

Generation of Einstein-Podolsky-Rosen State via Earth's Gravitational Field

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Abstract

Although various physical systems have been explored to produce entangled states involving electromagnetic, strong and weak interactions, the gravity has not yet been touched in practical entanglement generation. Here, we propose an experimentally feasible scheme with current technology for generating spin entangled neutron pairs via the Earth's gravitational field. The scheme is realized by passing two neutrons through a specific rectangular cavity, where the gravity adjusts the neutrons into entangled state. This provides a simple and practical way for the implementation of the test of quantum nonlocality and statistics in gravitational field.

Ever since the discovery of Bell inequalities [1], the generation of entanglement with various physical systems has been the intensively studied subject. Now the entanglement can be generated not only in optical [2, 3], atomic [4], solid state [5] systems where only the electromagnetic interactions is involved, but also with baryons [6], leptons [7], and mesons [8, 9, 10, 11] where the strong or weak interaction emerges as the dominant force. Many of these entanglement generation schemes have been experimentally realized in testing the violation of Bell's inequalities which reveal the nonlocality of quantum theories. Some of them have further found their roles in quantum computations and quantum information

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processing [12, 13, 14]. One considerable interest of exploring these various entangled systems is to show that the nonlocal correlation is not a peculiarity attributed to specific interactions but a universal quantum phenomena. Besides, such practical entangled systems contribute also as the crucial physical resource for quantum information science.

It has been noticed that three of the four fundamental interactions in nature, i.e., electromagnetic, strong and weak, are capable of generating entanglement leaving the gravity a sole exception [9]. Although some quantum effects of the classical gravity have been observed, i.e., quantum interference [15], discrete energy levels of neutrons in the gravitational potential [16], there is still no report on quantum entanglement generation concerning the Earth's gravitational field. In the above two cases, the neutrons act as a messenger revealing quantum effects of gravity. Moreover, the neutrons have long been considered the ideal tool to test the Bell inequalities [17]. However, due to the extremely small neutron-neutron scattering length ($\sim 10^{-14}$ m [18]), the usual way to generate entangled neutron pairs through their low energy scattering is a considerable technical challenge. Up to now, only the entanglement among different degree of freedoms of a single neutron has been experimentally realized [19, 20].

Here, we propose a scheme to generate the Einstein-Podolsky-Rosen state via Earth's gravitational field. The scheme composed of three different functional components: an energy filter that monochromatizes the neutrons, a rectangular cavity which entangles two neutrons, and the neutron spin polarization analyzers. The main idea is to guide a pair of monochromatic neutrons into the cavity where they are enforced into the same energy state. The spin singlet state should then be obtained due to the antisymmetrization requirement of the identical fermions. We design a practical experimental setup for our scheme, by which we show that, with the current technology, the gravity dominant entanglement generation and the nonlocality test with entangled neutron pairs are within the experimental reach.

As has been observed in [16], neutrons falling towards a horizontal reflecting mirror will distributed discontinuously in the vertical direction. Such a system can be described by the quantum theory of a particle bouncing in the gravitational field above a perfect mirror [21]. The Schrödinger equation governing the motion of the neutrons in the vertical dimension

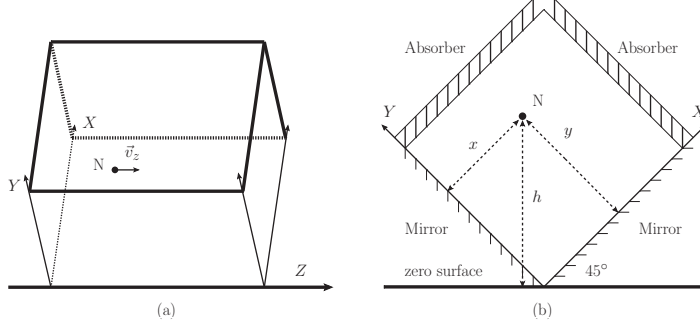


Figure 1: A rectangular cavity where the two lower surfaces are neutron mirrors and the two upper surfaces are neutron absorbers. (a): A neutron N passes through the rectangular cavity with longitudinal velocity \vec{v}_z ; (b): The transection of the rectangular cavity, h is the height of the neutrons relative to the zero gravitational potential surface.

reads

$$-\frac{\hbar^2}{2m} \frac{d}{d\xi^2} \phi(\xi) + V(\xi) \phi(\xi) = E \phi(\xi), \quad V(\xi) = \begin{cases} mg\xi & \xi \geq 0 \\ +\infty & \xi < 0 \end{cases}. \quad (1)$$

Here ξ is the height of neutron from the horizontal mirror. Equation (1) can be solved and shows discrete energy eigenstates [22]

$$\phi_n(\xi) = \mathcal{N}_n \text{Ai}\left(\xi \frac{(2m^2g)^{1/3}}{\hbar^{2/3}} - E_n \frac{2^{1/3}}{(mg^2\hbar^2)^{1/3}}\right), \quad (2)$$

where Ai are Airy functions and \mathcal{N}_n is a normalization constant, $E_n = \alpha_n \frac{(mg^2\hbar^2)^{1/3}}{2^{1/3}}$ corresponds to the energy eigenvalue of state ϕ_n with α_n being the n th zero of Airy function $\text{Ai}(-\alpha_n) = 0$.

Now considering a specifically designed rectangular cavity of Fig. 1, where the two lower surfaces of the cavity are neutron mirrors while the two upper surfaces are neutron absorbers. For each neutron in this cavity, it is subjected to a constant gravitational force $F = mg$, where m is the mass of neutron, g is the acceleration constant near the Earth's surface. Together with the lower mirrors, the Earth's gravitational field provides the confining potential well for the neutrons, which is

$$V(x, y) = \begin{cases} mg(x + y)/\sqrt{2} & x \geq 0 \text{ and } y \geq 0 \\ +\infty & x < 0 \text{ or } y < 0 \end{cases}, \quad (3)$$

where we have chosen the zero potential surface in Fig. 1. In the transection of the cavity, the X - Y plane of Fig. 1(b), the Schrödinger equation of motion for neutron takes the following form

$$-\frac{\hbar^2}{2m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi(x, y) + V(x, y) \Psi(x, y) = E \Psi(x, y) . \quad (4)$$

The energy eigenstates and eigenvalues of this equation can be similarly obtained as that of equation (1), and can be simply formulated as

$$\Psi_{n,m}(x, y) = \psi_n(x) \psi_m(y) , \quad E_{n,m} = E_n + E_m . \quad (5)$$

Here, $\psi_n(x) = \mathcal{N}_n \text{Ai}(x/l_0 - E_n/\varepsilon_0)$, \mathcal{N}_n is the normalization constant; l_0 and ε_0 are the characteristic length and energy defined as

$$l_0 = \hbar^{2/3} / (\sqrt{2} m^2 g)^{1/3} \simeq 6.59 \cdot 10^{-6} \text{ m} , \quad (6)$$

$$\varepsilon_0 = \sqrt[3]{m g^2 \hbar^2 / 4} \simeq 4.78 \cdot 10^{-13} \text{ eV} ; \quad (7)$$

and $E_n = \alpha_n \varepsilon_0$ is the eigen energy of ψ_n with α_n being defined after equation (2). The first four lowest eigenstates are $\Psi_{0,0}$, $\Psi_{0,1}$, $\Psi_{1,0}$, and $\Psi_{1,1}$, with $|\Psi_{n,m}(x, y)|^2$ being the value of the neutron distribution densities in the X - Y plane, see Fig. 2. Their corresponding energies can be listed as follows

$$E_{0,0} = E_0 + E_0 = \alpha_0 \varepsilon_0 + \alpha_0 \varepsilon_0 \simeq 2.22 \times 10^{-12} \text{ eV} , \quad (8)$$

$$E_{0,1} = E_0 + E_1 = \alpha_0 \varepsilon_0 + \alpha_1 \varepsilon_0 \simeq 3.06 \times 10^{-12} \text{ eV} , \quad (9)$$

$$E_{1,0} = E_1 + E_0 = \alpha_1 \varepsilon_0 + \alpha_0 \varepsilon_0 \simeq 3.06 \times 10^{-12} \text{ eV} , \quad (10)$$

$$E_{1,1} = E_1 + E_1 = \alpha_1 \varepsilon_0 + \alpha_1 \varepsilon_0 \simeq 3.90 \times 10^{-12} \text{ eV} , \quad (11)$$

where $E_{0,0}$ is the ground state energy and $E_{0,1}$ and $E_{1,0}$ are energies of the first degenerated excited states.

From the density distributions $|\Psi_{n,m}(x, y)|^2$ plotted in Fig. 2, we see that the quantum states of higher energies become densely distributed in the area with larger values of x , y . If the two upper absorbers are set at a position of $x = y \simeq 3l_0$, the transection of rectangular cavity would have the size holding only the ground state. In this configuration, where the excited states $\Psi_{0,1}$, $\Psi_{1,0}$, $\Psi_{1,1}$ and other even higher excited states are substantially absorbed

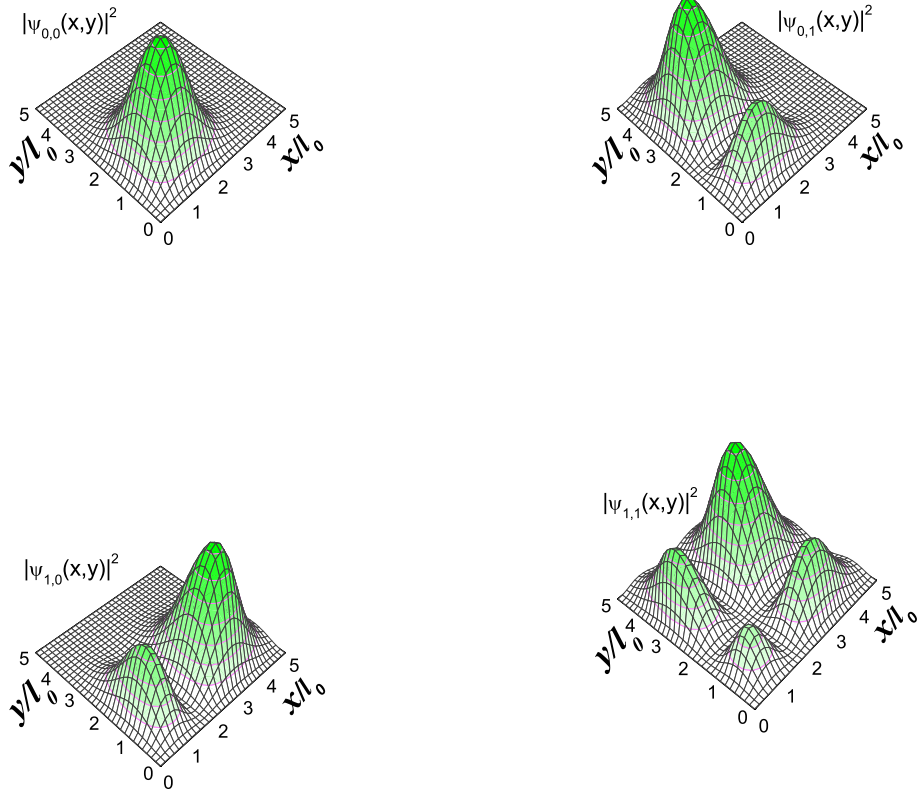


Figure 2: The probability distributions of the first four lowest eigenstates in the transection of the rectangular cavity: $|\Psi_{0,0}|^2$, $|\Psi_{0,1}|^2$, $|\Psi_{1,0}|^2$, $|\Psi_{1,1}|^2$. The X and Y axes are in unit of l_0 .

while the neutron transmitting in the cavity, only the ground state may survive. The minimal time Δt while the neutron has to stay in the cavity to resolve different quantum states is [22]

$$\Delta t = \hbar/(E_{0,1} - E_{0,0}) \sim 7.84 \cdot 10^{-4} \text{s} . \quad (12)$$

After a significantly longer time duration $T \gg \Delta t$ in the cavity, a substantial absorbtion of the excited state will be expected in the present configuration of $x = y \simeq 3l_0$.

The neutrons from the same monochromatic source having the the same energy spectrum and velocity will have the same uncertainty in momentum, the Δp_z . The Heisenberg uncer-

tainty relation ascertains the indistinguishability of the neutron in longitudinal position z as: $\Delta z \Delta p_z \geq \hbar/2$. For slow neutrons $\Delta p_z = m \Delta v_z$, we can define the full width at half maximum (FWHM) of the velocity spectrum $\overline{\Delta v_z}$ whose relation with Δv_z can be estimated via the minimum uncertainty wave packet: $\overline{\Delta v_z} = 2\sqrt{2 \ln 2} \cdot \Delta v_z$. Thus we have $\Delta z \overline{\Delta v_z} > \hbar/m$. Two neutrons A and B will keep a relative displacement of $\delta z = T \delta v_z = T |v_z^{(A)} - v_z^{(B)}|$ through the passage in the cavity. In case the relative displacement of neutrons $\delta z < \Delta z$, i.e., $\delta z < \hbar/(m \overline{\Delta v_z})$, while they enter the cavity “simultaneously”, two neutrons will stay longitudinally indistinguishable when coming out the cavity. If we require the neutrons with relative velocity $\delta v_z \leq 2 \cdot \overline{\Delta v_z}$ to be indistinguishable, the following relation must be satisfied

$$(\overline{\Delta v_z})^2 < \frac{\hbar}{2mT} . \quad (13)$$

This gives a quantitative condition for two neutrons to be longitudinally indistinguishable throughout the cavity. At the same time, in the transection plane, the two neutrons would be projected into the same energy state $\Psi_{0,0}(x, y)$ while approaching the exit of the cavity as the excited states are absorbed by the two upper absorbers, that is

$$\Psi^{(AB)} = \Psi_{0,0}^{(A)} \Psi_{0,0}^{(B)} . \quad (14)$$

Equation (14) is symmetric under the permutation of the two neutrons. Due to the Fermi-Dirac statistics of identical fermions, if two neutrons enter the cavity simultaneously and equation (13) is satisfied, the spin wave function of the neutron pair must be antisymmetric, i.e.

$$\psi_s^{(AB)} = \frac{1}{\sqrt{2}}(|+ -\rangle - |- +\rangle) \quad (15)$$

when leaving the cavity. Inputting the values of \hbar , m , and energy $E_{0,0}$, $E_{0,1}$ into equation (13) and choose T to be two orders greater than Δt (i.e. $T = 10^2 \cdot \Delta t$), we get the critical value for $\overline{\Delta v_z}$ as

$$\overline{\Delta v_z} < 0.6 \cdot 10^{-3} \text{ ms}^{-1} . \quad (16)$$

Note that while choosing $T = 10 \cdot \Delta t$, we then get $\overline{\Delta v_z} < 2 \cdot 10^{-3} \text{ ms}^{-1}$. Thus the neutron sources with a FWHM of velocity less than 10^{-3} ms^{-1} are capable of generating spin entanglement within our scheme.

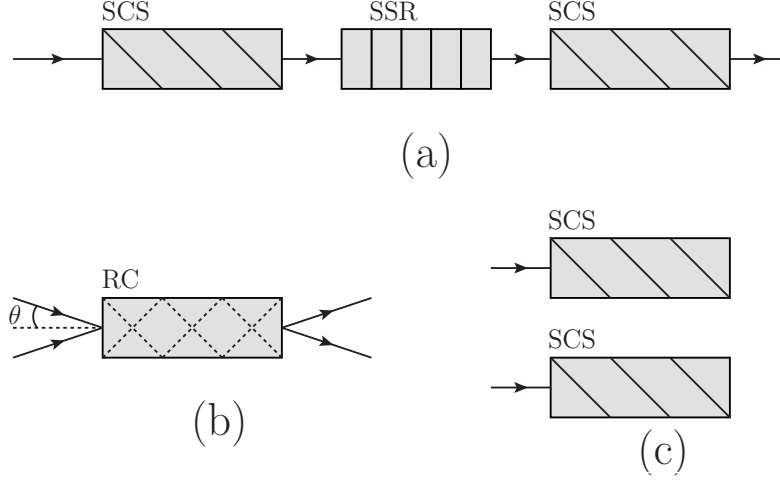


Figure 3: A schematic setup of the devices for the generation of gravity induced entangled neutron pairs and the measurement of Bell inequalities. Here the arrows represent the motion of neutron. (a) is the energy filter of neutrons, including: two superconducting solenoid polarizers (SCS) and a spatial spin resonance (SSR) unit; (b) is the top view of the rectangular cavity (RC) of Figure 1; (c) is a compound of two SCS acting as two separated neutron polarization analyzers.

In the following we will present a practical experiment setup for generating entangled neutron pairs via the Earth's gravitational field using ultra cold neutrons (UCN, velocity $v < 10 \text{ ms}^{-1}$ [23] and we choose $v = 5 \text{ ms}^{-1}$ for numerical evaluation hereafter). The three ingredients in our scheme which have been stated at the start of the paper include: the energy filter, the rectangular entangling cavity and the spin analyzers, shown in Figure 3.

First, the monochromatic UCN of $\overline{\Delta\lambda}/\lambda = \overline{\Delta v}/v < 10^{-3}$ (using values of equation (16)) could be achieved via a Drabkin energy filter [24] which is composed of a polarizer, spatial spin resonance units, and a polarization analyzer. Using the superconducting solenoid-polarizer [25] (the left SCS in Figure. 3(a)), the UCNs with different polarizations (parallel or antiparallel to magnetic field) have contrary behaviors: one polarization passes unhindered through the solenoid while the other one being reflected back. After this, the degree of polarization of UCN here can reach the level of 100% [26]. The spatial spin resonance units (SSR in Fig. 3(a)) only flip the spin of neutrons for specific wavelength (velocity) [24]. The spin analyzer (the right SCS in Fig. 3(a)) selects the spin flipped neutrons which now have the monochromatic wavelength. A precision of $\overline{\Delta\lambda}/\lambda \simeq 10^{-2}$ has already been reached in [27] for neutrons with $\lambda = 5 \text{ \AA}$. As the resolution of the λ is inversely proportional to

the number of SSR units that neutrons pass through, an improvement to less than 10^{-3} is technically straight forward [28].

After the monochromatization, two neutrons are led to the rectangular cavity as Fig. 1 with relative angle of $\theta \sim 10^{-2}$ (see Fig. 3(b)) in the horizontal plane ensuring their velocity in the transection plane is about 10^{-2} ms^{-1} . The rectangular cavity with a length of about $(5\text{ms}^{-1}) \cdot T$ would be enough for two neutrons coming out the cavity to be in spin singlet state. The out going neutrons can be guided to sufficient long distance as the depolarization effect of UCN in collision with materials is quite low, $\sim 10^{-5}$ per collision [29]. Localized polarization analyzers can then be applied to the two well separated neutrons (see Fig. 3(c)). In particular, when two analyzers are placed in parallel, if one neutron passes through the solenoid polarizer then we can determine with certainty that the other neutron would be bounced backward from its solenoid polarizer though there is no interaction to project it to specific polarization state. A measurement of the Bell inequalities can be acquired by actively choosing different directions of the two solenoid polarizers separately.

In conclusion, it is demonstrated that through a particular type of rectangular cavity two neutrons immersed in the earth's gravitational field will entangle with each other. This enables us to test the quantum nonlocality involving the gravity which is the only fundamental interaction of nature having not yet been touched in practical entanglement generation so far. Due to the high detection efficiency of massive particle and the manageable large spatial separation between two neutrons (mean life of neutron at rest is $\tau \sim 885.7\text{s}$ [30]), the proposed scheme in this work provides a simple and practical way for the implementation of nonlocality test of quantum entanglement and statistics in gravitational field, while a more conclusive test of local hidden variables theory would also be expected.

Acknowledgments

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Appendix

Separation of the variables

Considering the potential of equation (3) in the region of $x \geq 0$, and $y \geq 0$, equation (4) can be expressed as

$$\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{mgx}{\sqrt{2}}\right)\Psi(x, y) + \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{mgy}{\sqrt{2}}\right)\Psi(x, y) = E\Psi(x, y) \quad (17)$$

Defining $\Psi(x, y) = \psi(x)\psi(y)$, the separation of variables, and $E = E_x + E_y$, the above equation can then be expressed as

$$\begin{cases} \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{mgx}{\sqrt{2}}\right)\psi(x) = E_x\psi(x) \\ \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{mgy}{\sqrt{2}}\right)\psi(y) = E_y\psi(y) \end{cases} \quad (18)$$

These two equations can be solved in a similar way as equation (2). The solutions are

$$\psi_n(x) = \mathcal{N}_n \text{Ai}(x/l_0 - E_n/\varepsilon_0), \quad \psi_m(y) = \mathcal{N}_m \text{Ai}(y/l_0 - E_m/\varepsilon_0),$$

where Ai are Airy functions; $n, m \in \{0, 1, 2, \dots\}$; \mathcal{N}_n is the normalization constant; l_0, ε_0 are the characteristic length and energy defined as $l_0 = \hbar^{2/3}/(\sqrt{2}m^2g)^{1/3}$, $\varepsilon_0 = \sqrt[3]{mg^2\hbar^2/4}$.

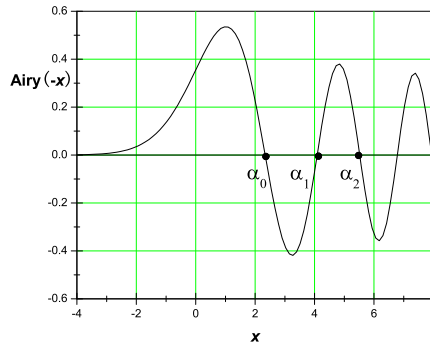


Figure 4: The zeros of Airy function: $\text{Ai}(-\alpha_n) = 0$, $n \in \{0, 1, 2, \dots\}$.

The eigenvalues of energy can be obtained by imposing the boundary condition $\psi_n(0) = \mathcal{N}_n \text{Ai}(-E_n/\varepsilon_0) = 0$. We can get $E_n = \alpha_n \varepsilon_0$, where α_n is the n th zero of Airy function, see Figure 4.

Bouncing in the cavity

In the transection plane of the cavity, the ground state in the gravitational potential is $\Psi_{0,0}(x, y) = \psi_0(x)\psi_0(y)$. Inputting the numerical values E_0, \mathcal{N}_0 into the wave function we have

$$\Psi_{0,0}(x, y) \simeq \frac{25}{81} \text{Ai}(x/6.59 - 2.34) \cdot \text{Ai}(y/6.59 - 2.34), \quad (19)$$

$$E_{0,0} = 2.22 \cdot 10^{-12} \text{ eV}, \quad (20)$$

where x, y are in unit of $\mu\text{m} = 10^{-6}\text{m}$. In the ground state, $\langle x \rangle = \langle y \rangle \simeq 10.29 \mu\text{m}$, $\langle x^2 \rangle = \langle y^2 \rangle \simeq 126.89 (\mu\text{m})^2$, then $\Delta x = \Delta y = 4.59 \mu\text{m}$. Due to the Heisenberg uncertainty relation $\Delta x \Delta p_x \geq \hbar/2$ and $p_x = mv_x$, we have $\Delta v_x \geq 6.86 \cdot 10^{-3} \text{ ms}^{-1}$. The maximum velocity in x axis is $v_{x\text{max}} = \sqrt{2E_0/m} \simeq 1.46 \cdot 10^{-2} \text{ ms}^{-1}$. The average velocity \bar{v}_x in x axis satisfies

$$\bar{v}_x = \frac{v_{x\text{max}}}{2} \lesssim \Delta v_x. \quad (21)$$

The number of bouncing times of the neutron in the cavity is $n = \frac{T \cdot \bar{v}_x}{2l_x}$, where l_x is the side length of the transection plane. The uncertainty of the bouncing times n arises in regard of the variance Δv_x , and can be expressed as $\Delta n = \frac{T \Delta v_x}{2l_x} \gtrsim n$, which tells $\Delta n \geq 1$. Consequently, it is not distinguishable whether the neutron bounces in the cavity with odd or even number of times.

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